Electromagnetic Dyadic Green's Functions for Multilayered Spheroidal Structures—I: Formulation

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***Abstract*—Dyadic Green's functions (DGFs) and their scattering coefficients are formulated in this paper for defining the electromagnetic fields in multilayered spheroidal structures. The principle of scattering superposition is applied. In a similar form of the DGF in an unbounded medium under spheroidal coordinates, the scattering DGFs due to multiple spheroidal interfaces are expanded in terms of the spheroidal vector wave functions. For the lack of general orthogonality of the spheroidal radial and angular functions, the Green's dyadics are expressed in a different way where the coordinate unit vectors are also combined in the construction, as compared with the conventional form of vector wave eigenfunction expansion. The matrix equation systems satisfied by the coupled scattering (i.e., reflection and transmission) coefficients of the DGFs are obtained so that these coefficients can be solved uniquely. The DGFs can be employed to investigate effects of spheroidal radomes used to protect the airborne or satellite antenna systems and of handy phone radiation near the spheroid-shaped human head, and so forth. Numerical calculations about the applications of the formulated multilayered DGFs will be presented in part II of this paper.**

***Index Terms*—Antenna radiation, dyadic Green's function, electromagnetic-wave theory, spheroidal wave functions, stratified media.**

I. Introduction

A SPHEROIDAL structure, as a very common geometry, has been widely used to realistically model practical problems such as spheroidal airborne antenna radome and the human head. Usually, two classes of problems are of great interest and/or concerns. One is the electromagnetic (EM) *scattering* associated with dielectric spheroidally stratified media, and the other is the EM *radiation* in spheroidally multilayered structures.

For the former, a series of works has been carried out to date about plane EM waves scattered by a single spheroid [1], [2], and a system of spheroids [3], [4]. For the latter, only a very little amount of work has been reported thus far [5]. EM scattering can, in general, be regarded as the EM radiation from a point source of varying current distribution with the space distance that is located at infinity. When the source is located inside an intermediate region of stratified spheroidal structure, the multiple scattering is involved and the wave modes inside the region become more complicated. In this sense, the EM radiation problem is more general as compared with the EM scattering problems.

To analyze EM radiated fields in spheroidal geometries, the dyadic Green's function (DGF) technique provides a straightforward way. The DGFs in various geometries such as single stratified planar, cylindrical, and spherical structures were formulated [6], [7]. In multilayered geometries, the DGFs have also been constructed and their coefficients derived. Usually, two types of DGFs, i.e., the EM DGFs and the Hertzian vector potential DGFs, were expressed. Three methods that are available in the literature, i.e., the Fourier transform technique (normally in planar structures only), wave matrix operator, and/or transmission line (frequently in planar structures) methods, and vector wave eigenfunction expansion method (in regular structures where vector wave functions are orthogonal) were developed.

In a planar stratified geometry [6], Lee and Kong [8] employed Fourier transform to deduce the DGFs in an anisotropic medium, Sphicopoulos *et al.* [9] used an operator approach to derive the DGFs in isotropic and achiral media, Das and Pozer [10] utilized the Fourier transform technique, Vegni *et al.* [11], and Nyquist and Kzadri [12] made use of wave matrices in the electric Hertz potential to obtain the DGFs and their scattering coefficients in isotropic and achiral media, Pan and Wolff [13] employed scalarized formulas, Dreher [14] used the Fourier transform and method of lines to re-derive the DGFs and their coefficients in the same media, Mesa *et al.* [15] applied the equivalent boundary method to obtain the DGFs and their coefficients in two-dimensional (2-D) inhomogeneous bianisotropic media, Ali *et al.* [16] used the Fourier transform and Li *et al.* [17] employed the vector wave eigenfunction expansion to formulate the DGFs and formulated their coefficients in isotropic and chiral media, Bernardi *et al.* [18] again employed Fourier transform and operator technique to the same medium, but with backed conducting ground plane, Barkeshli utilized the Fourier transform technique to express the DGFs and their coefficients in anisotropicuniaxial media [19], dielectric/magnetic media [20], and gyroelectric media [21], and Habashy *et al.* [22] applied the Fourier transform technique to work out the DGFs in arbitrarily magnetizedlinear plasma. For the cases of a free-space (or unbounded space), a single-layered

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medium or multilayered structure many references exist, such as various representations by Pathak [23], Cavalcante *et al.* [24], Engheta and Bassiri [25], Chew [7], Glisson and Junker [26], Krowne [27], Lakhtakia [28], and Toscano and Vegni [29], and Weiglhofer [30], [31]. Since there is a large number of publications available, it is impractical to list all of them here.

In a multilayered cylindrical geometry [6], the DGFs in the `chiral` media and the specific coefficients were given by Yin and Wang [32]. The unified DGFs in `chiral` media and their scattering coefficients in general form were formulated by Li *et al.* [33].

In a multilayered spherical geometry [6], [34], [35], the DGFs in `achiral` media and their scattering coefficients were generalized by Li *et al.* [36]. This work was extended later to the DGFs in `chiral` media by Li *et al.* [37].

In a spheroidal geometry, the DGFs in an unbounded medium were constructed in 1995 by Giarola [38] and Li *et al.* [39], respectively. Also, the scattering DGFs in the presence of: 1) a perfectly conducting prolate spheroid [38] and 2) a dielectric spheroid that can reduce to a conducting spheroid by letting the permittivity to approach infinity [39] were represented. It is shown in [39] that the formulating of the DGFs in spheroidal structures is difficult and the difficulty is due to the following two issues: 1) no recursive relations of the spheroidal angular and radial functions can be obtained by the methods usually used for the more common special functions of mathematical physics (the existing recurrence relations of Whittaker type are, as stated by Meixner [40], actually identities, not the recursion formulas) and 2) the coupling series coefficients of the scattered fields must be numerically calculated by inversion of coefficients of matrices.

This paper, as an extension of previous work [39], represents the DGFs in a multilayered spheroidal structure and their scattering coefficient matrices in general form. Multiple reflections and transmissions are considered in the construction of the scattering DGFs. Various possibilities that the source distribution and observation point are, respectively, located in an arbitrarily assumed region of the multilayered structure are considered in the formulation. The matrix equation system satisfied by the coupled scattering coefficients from the boundary conditions at the spheroidal interfaces are obtained and solved.

II. Fundamental Formulation

To analyze the EM fields in spheroidal structures, we consider a prolate spheroidal geometry of multilayers, as shown in Fig. 1. Here, η is an angular coordinate (ranged within $-1 \le \eta \le 1$), ξ is a radial one (ranged within $1 \le \xi < \infty$), ϕ is an azimuthal one (ranged within $0 \le \phi < 2\pi$), and each spheroidal interface is assumed to have the same interfocal distance d. The relations between the prolate spheroidal coordinates and the rectangular coordinates are given as follows [40]:

$$x = \frac{d}{2}\sqrt{(1-\eta^2)(\xi^2-1)}\cos\phi \tag{1a}$$

$$y = \frac{d}{2}\sqrt{(1-\eta^2)(\xi^2-1)}\sin\phi \tag{1b}$$

$$z = \frac{d}{2}\eta\xi. \tag{1c}$$

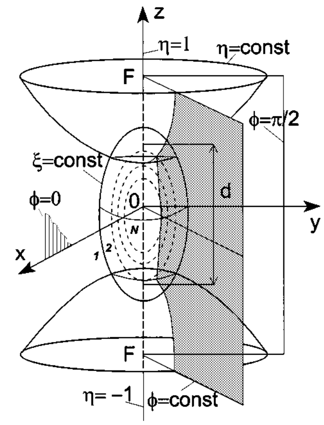

Fig. 1. Geometry of a multilayered prolate spheroid under coordinates (ξ, η, ϕ).

Oblate spheroidal problems can be analyzed by a similar procedure presented here or by the symbolic transformation $\xi \rightarrow \pm i\xi$ and $c \rightarrow \mp ic$, where $c = (1/2)kd$ (k is the wave propagation constant). The ranges of η and ξ in the oblate spheroidal system belong to $0 \le \eta \le 1$ and $-\infty \le \xi < \infty$, respectively.

Assume that the space is divided by $N-1$ spheroidal interfaces into N regions, as shown in Fig. 1. The spheroidally stratified regions are labeled, respectively, as $1, 2, 3, \ldots$, and N. The EM radiated fields $\boldsymbol{E}_f$ and $\boldsymbol{H}_f$ in the field (fth) region ($f = 1, 2, 3, \ldots$, and N) due to the electric and magnetic current distributions $\boldsymbol{J}_s$ and $\boldsymbol{M}_s$ located in the source (sth) region ($s = 1, 2, 3, \ldots$, and N), as shown in Fig. 1, can be expressed by

$$\nabla\times\nabla\times\boldsymbol{E}_f - k_f^2\boldsymbol{E}_f = \left[i\omega\mu_f\boldsymbol{J}_f - (\nabla\times\boldsymbol{M})_f\right]\delta_{fs}, \tag{2a}$$

$$\nabla\times\nabla\times\boldsymbol{H}_f - k_f^2\boldsymbol{H}_f = \left[i\omega\varepsilon_f\boldsymbol{M}_f + (\nabla\times\boldsymbol{J})_f\right]\delta_{fs}, \tag{2b}$$

where δ_{fs} denotes the Kronecker delta (=1 for $f = s$ and 0 for $f \ne s$), $k_f = \omega\sqrt{\mu_f\varepsilon_f(1+(i\sigma_f/\omega\varepsilon_f))}$ is the wave propagation constant in the fth layer of the multilayered medium, and ε_f, μ_f, and σ_f identify the permittivity, permeability, and conductivity of the medium, respectively. The subscript (fs) denotes the layers where the field point and the source point are located, respectively. A time dependence $\exp(-i\omega t)$ is assumed to describe the EM fields throughout this paper.

The EM fields excited by an electric current source $\boldsymbol{J}_s$ and a magnetic current distribution $\boldsymbol{M}_s$ can be expressed in terms of integrals containing DGFs as follows [6], [33], [36]:

$$\boldsymbol{E}_f(\boldsymbol{r}) = i\omega\mu_f \iiint_V \overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')\cdot\boldsymbol{J}_s(\boldsymbol{r}')\,dV' - \iiint_V \overline{\boldsymbol{G}}_{\mathrm{EM}}^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')\cdot\boldsymbol{M}_s(\boldsymbol{r}')\,dV' \tag{3a}$$

$$\boldsymbol{H}_f(\boldsymbol{r}) = i\omega\varepsilon_f \iiint_V \overline{\boldsymbol{G}}_{HM}^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')\cdot\boldsymbol{M}_s(\boldsymbol{r}')\,dV' + \iiint_V \overline{\boldsymbol{G}}_{HJ}^{(fs)}(\boldsymbol{r}, \boldsymbol{r}')\cdot\boldsymbol{J}_s(\boldsymbol{r}')\,dV' \tag{3b}$$

where the prime denotes the coordinates (ξ', η', ϕ') of the current sources $\boldsymbol{J}_s$ and $\boldsymbol{M}_s$, and V identifies the volume occupied by the sources in the second region, the superscript (fs) denotes the layers where the field point and the source point are located, respectively.

Substituting (3a) and (3b) into (2a) and (2b) respectively, we obtain

$$\nabla\times\nabla\times\begin{bmatrix}\overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')\\ \overline{\boldsymbol{G}}_{HM}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')\end{bmatrix}-k_f^2\begin{bmatrix}\overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')\\ \overline{\boldsymbol{G}}_{HM}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')\end{bmatrix}=\overline{\boldsymbol{I}}\delta(\boldsymbol{r}-\boldsymbol{r}')\delta_{fs} \tag{4a}$$

$$\nabla\times\nabla\times\begin{bmatrix}\overline{\boldsymbol{G}}_{HJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')\\ \overline{\boldsymbol{G}}_{\mathrm{EM}}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')\end{bmatrix}-k_f^2\begin{bmatrix}\overline{\boldsymbol{G}}_{HJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')\\ \overline{\boldsymbol{G}}_{\mathrm{EM}}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')\end{bmatrix}=\nabla\times\overline{\boldsymbol{I}}\delta(\boldsymbol{r}-\boldsymbol{r}')\delta_{fs} \tag{4b}$$

where $\overline{\boldsymbol{I}}$ stands for the unit/identity dyad and $\delta(\boldsymbol{r}-\boldsymbol{r}')$ identifies the Dirac delta function. Tai [6] defined $\overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ and $\overline{\boldsymbol{G}}_{HJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ as the electric and magnetic DGFs of the first kind—$\overline{\boldsymbol{G}}_{e1}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ and $\overline{\boldsymbol{G}}_{m1}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$, and $\overline{\boldsymbol{G}}_{\mathrm{EM}}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ and $\overline{\boldsymbol{G}}_{HM}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ as the electric and magnetic DGFs of the second kind—$\overline{\boldsymbol{G}}_{e2}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ and $\overline{\boldsymbol{G}}_{m2}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$.

Since $\overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ and $\overline{\boldsymbol{G}}_{HJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ are related by the first elements of (4a) and (4b), while $\overline{\boldsymbol{G}}_{\mathrm{EM}}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ and $\overline{\boldsymbol{G}}_{HM}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ are related by the second elements of (4a) and (4b), we do not need to derive all of them. Therefore, only the formulations of $\overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ and $\overline{\boldsymbol{G}}_{HM}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ will be considered. The following boundary conditions at the spheroidal interface $\xi=\xi_f$ are satisfied by various types of DGFs after (3a) and (3b) is substituted into the Dirichlet boundary conditions

$$\hat{\xi}\times\begin{bmatrix}\overline{\boldsymbol{G}}_{EJ}^{(fs)}\\ \overline{\boldsymbol{G}}_{HM}^{(fs)}\end{bmatrix}=\hat{\xi}\times\begin{bmatrix}\overline{\boldsymbol{G}}_{EJ}^{[(f+1)s]}\\ \overline{\boldsymbol{G}}_{HM}^{[(f+1)s]}\end{bmatrix} \tag{5a}$$

$$\begin{bmatrix}1/\mu_f\\ 1/\epsilon_f\end{bmatrix}\hat{\xi}\times\nabla\times\begin{bmatrix}\overline{\boldsymbol{G}}_{EJ}^{(fs)}\\ \overline{\boldsymbol{G}}_{HM}^{(fs)}\end{bmatrix}=\begin{bmatrix}1/\mu_{f+1}\\ 1/\epsilon_{f+1}\end{bmatrix}\hat{\xi}\times\nabla\times\begin{bmatrix}\overline{\boldsymbol{G}}_{EJ}^{[(f+1)s]}\\ \overline{\boldsymbol{G}}_{HM}^{[(f+1)s]}\end{bmatrix} \tag{5b}$$

where $\left[\begin{smallmatrix}1/\mu_{f+1}\\ 1/\epsilon_{f+1}\end{smallmatrix}\right]$ stands for the ruling that either the upper elements or the lower elements of the matrices should be taken at the same time. In fact, (5a) and (5b) represent four equations if all the upper and lower elements are considered, respectively.

Furthermore, the DGF $\overline{\boldsymbol{G}}_{HM}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ can be obtained from the $\overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ by making the simple duality replacements $\boldsymbol{E}\to\boldsymbol{H}$, $\boldsymbol{H}\to-\boldsymbol{E}$, $\boldsymbol{J}\to\boldsymbol{M}$, $\boldsymbol{M}\to-\boldsymbol{J}$, $\mu\to\varepsilon$, and $\varepsilon\to\mu$. In this paper, only the DGF $\overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r},\boldsymbol{r}')$ is represented to avoid unnecessary repetition.

III. Unbounded DGFs

A. Method of Separation of Variables

According to Collin [41], the scalar Green's function $g(\boldsymbol{r},\boldsymbol{r}')$ satisfies the following differential equation:

$$(\nabla^2+k^2)g(\boldsymbol{r},\boldsymbol{r}')=-\delta(\boldsymbol{r}-\boldsymbol{r}'). \tag{6}$$

In a source-free region, the solution of the EM fields E_{mn} and H_{mn} of the wave modes mn can be found by using the well-known method of separation of variables, and is given by the radial function $\mathcal{H}(k,\xi)$ and the angular functions $\Theta(k,\eta)$ and $\Phi(k,\phi)$ as follows:

$$\begin{aligned}\mathcal{H}(k,\xi)&=A\mathcal{P}_n^m(c,\xi)+B\mathcal{L}_n^m(c,\xi)\\&=A'R_{mn}^{(1)}(c,\xi)+B'R_{mn}^{(2)}(c,\xi)\end{aligned} \tag{7a}$$

$$\begin{aligned}\Theta(k,\eta)&=C\mathcal{P}_n^m(c,\eta)+D\mathcal{L}_n^m(c,\eta)\\&=C'S_{mn}^{(1)}(c,\eta)+D'S_{mn}^{(2)}(c,\eta)\end{aligned} \tag{7b}$$

$$\Phi(k,\phi)=E\cos(m\phi)+F\sin(m\phi) \tag{7c}$$

where m and n identify the eigenvalue parameters, A, B, A', B', C, D, C', D', E, and F are constants, and $\mathcal{P}_n^m(\alpha,\beta)$ and $\mathcal{L}_n^m(\alpha,\beta)$ denote the generalized Legendre functions in general [42].

However, $\mathcal{P}_n^m(c,\xi)$ and $\mathcal{L}_n^m(c,\xi)$ are referred to as the first and second kinds of radial functions $R_{mn}^{(1)}(c,\xi)$ and $R_{mn}^{(2)}(c,\xi)$ [40], respectively. They can also be considered as the generalized spherical Bessel functions of the first and second kinds since they have the similar properties as compared to $j_n(kr)$ and $y_n(kr)$ in spherical coordinates. Therefore, the third and fourth kinds of radial functions $R_{mn}^{(3)}(c,\xi)$ and $R_{mn}^{(4)}(c,\xi)$ can also be constructed in terms of the first and second kinds, similar to those of the third and fourth kinds of spherical Bessel functions (i.e., the Hankel functions $h_n^{(1)}(kr)$ and $h_n^{(2)}(kr)$ of the first and second kinds). To simplify the representation of radial functions of different kinds, the radial function of the ith kind, $R_{mn}^{(i)}(c,\xi)$ ($i=1,2,3,$ and 4) takes the usual form. In a similar form of the associated Legendre function $P_n^m(\eta)$ in the spherical case, the angular function for a spheroidal case is chosen as $S_n^m(c,\eta)$ [40].

Thus, the scalar wave eigenfunctions are given by [40]

$$\psi_{\substack{e\\o}mn}^{(i)}(c,\boldsymbol{r})=R_{mn}^{(i)}(c,\xi)S_n^m(c,\eta)\begin{matrix}\cos\\ \sin\end{matrix}(m\phi) \tag{8}$$

where, for the fields inside the spheroid, the first kind of radial function ($i=1$) is taken and for the fields outside the spheroid, the third kind ($i=3$) is used because of the time dependence chosen. For the intermediate region between the two spheroidal interfaces, both the first and third kinds of the radial functions are used in the construction of the DGFs.

B. Unbounded Scalar Green's Function

In terms of the above scalar spheroidal wave functions, the scalar Green's function has been formulated [40] and is given by

$$\begin{aligned}g(\boldsymbol{r},\boldsymbol{r}')&=\frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\\&=\frac{ik}{2\pi}\sum_{n=m}^{\infty}\sum_{m=0}^{\infty}\frac{2-\delta_{m0}}{N_{mn}}\psi_{\substack{e\\o}mn}^{(3)}(c,\boldsymbol{r}^{>})\psi_{\substack{e\\o}mn}^{(1)}(c,\boldsymbol{r}^{<})\end{aligned}$$

$$= \frac{ik}{2\pi} \sum_{n=m}^{\infty} \sum_{m=0}^{\infty} \frac{2-\delta_{m0}}{N_{mn}} S_n^m(c,\,\eta) \cdot S_n^m(c,\,\eta') \frac{\cos}{\sin}\left[m(\phi-\phi')\right] \cdot \begin{cases} R_{mn}^{(3)}(c,\,\xi) R_{mn}^{(1)}(c,\,\xi'), & \xi \ge \xi' \\ R_{mn}^{(1)}(c,\,\xi) R_{mn}^{(3)}(c,\,\xi'), & \xi \le \xi' \end{cases} \tag{9}$$

where $\boldsymbol{r}^{>}$ and $\boldsymbol{r}^{<}$ denote the coordinate vector $\boldsymbol{r}$, where ξ is taken as $\max(\xi,\,\xi')$ and $\min(\xi,\,\xi')$, respectively, while the coordinates η, ϕ and η', ϕ' should be adopted correspondingly; δ_{m0} is the Kronecker delta and N_{mn} is the normalization factor of the angular function of the first kind.

C. Unbounded Green's Dyadics

To formulate the DGFs, one way is to solve (4a) for them, and the other is to employ the following relations between the Green's dyadics and scalar Green's function in the unbounded space, according to Tai [6] and Collin [41]

$$\overline{\boldsymbol{G}}_{EJ0}(\boldsymbol{r},\,\boldsymbol{r}') = \left[1 + \frac{1}{k^2}\nabla\nabla\cdot\right]\left[\overline{\boldsymbol{I}} g(\boldsymbol{r},\,\boldsymbol{r}')\right] \tag{10a}$$

$$\overline{\boldsymbol{G}}_{HJ0}(\boldsymbol{r},\,\boldsymbol{r}') = \nabla\times\left[\overline{\boldsymbol{I}} g(\boldsymbol{r},\,\boldsymbol{r}')\right] = \nabla g(\boldsymbol{r},\,\boldsymbol{r}')\times\overline{\boldsymbol{I}} \tag{10b}$$

where the additional subscript 0 beside EJ and HJ stands for the unbounded space.

In terms of the above-defined spheroidal vector wave functions, in explicit bi-vector form, the electric DGFs given in (10a) can be obtained after substitution of (9) for $\xi \gtrless \xi'$ as

$$\begin{aligned}\overline{\boldsymbol{G}}_{EJ0}(\boldsymbol{r},\,\boldsymbol{r}') = & -\frac{\hat{\xi}\hat{\xi}}{k_s^2}\,\delta(\boldsymbol{r}-\boldsymbol{r}') \\ & + \frac{i}{2\pi}\sum_{n=m}^{\infty}\sum_{m=0}^{\infty}\frac{2-\delta_{m0}}{N_{mn}}\begin{bmatrix}\psi_{\substack{e\\o}mn}^{(1)}(c,\,\boldsymbol{r}') \\ \psi_{\substack{e\\o}mn}^{(3)}(c,\,\boldsymbol{r}')\end{bmatrix} \\ & \cdot\left\{\begin{bmatrix}\boldsymbol{N}_{\substack{e\\o}mn}^{x(3)}(c,\,\boldsymbol{r}) \\ \boldsymbol{N}_{\substack{e\\o}mn}^{x(1)}(c,\,\boldsymbol{r})\end{bmatrix}\hat{\boldsymbol{x}} + \begin{bmatrix}\boldsymbol{N}_{\substack{e\\o}mn}^{y(3)}(c,\,\boldsymbol{r}) \\ \boldsymbol{N}_{\substack{e\\o}mn}^{y(1)}(c,\,\boldsymbol{r})\end{bmatrix}\hat{\boldsymbol{y}} + \begin{bmatrix}\boldsymbol{N}_{\substack{e\\o}mn}^{z(3)}(c,\,\boldsymbol{r}) \\ \boldsymbol{N}_{\substack{e\\o}mn}^{z(1)}(c,\,\boldsymbol{r})\end{bmatrix}\hat{\boldsymbol{z}}\right\}\end{aligned} \tag{11}$$

where $\hat{\xi}$ denotes the spheroidal radial unit vector, $\delta(\boldsymbol{r}-\boldsymbol{r}')$ is the three-dimensional Dirac delta function, and the prime denotes the coordinates $(\xi',\,\eta',\,\phi')$. The first term of (11) stands for the nonsolenoidal contribution and can be obtained by using the same method given by Tai [6, pp. 128–129, 154.]. The spheroidal vector wave functions $\boldsymbol{M}_{\substack{e\\o}mn}^{a(i)}(c,\,\boldsymbol{r})$ and $\boldsymbol{N}_{\substack{e\\o}mn}^{a(i)}(c,\,\boldsymbol{r})$ ($\hat{\boldsymbol{a}} = \hat{\boldsymbol{x}},\,\hat{\boldsymbol{y}},\,\hat{\boldsymbol{z}}$) for the construction of Green's dyadics are defined in terms of the above scalar eigenfunctions as follows:

$$\boldsymbol{M}_{\substack{e\\o}mn}^{a(i)}(c,\,\boldsymbol{r}) = \nabla\times\left[\psi_{\substack{e\\o}mn}^{(i)}(c,\,\boldsymbol{r})\hat{\boldsymbol{a}}\right], \qquad \hat{\boldsymbol{a}} = \hat{\boldsymbol{x}},\,\hat{\boldsymbol{y}},\,\hat{\boldsymbol{z}} \tag{12a}$$

$$\boldsymbol{N}_{\substack{e\\o}mn}^{a(i)}(c,\,\boldsymbol{r}) = \frac{1}{k}\nabla\times\nabla\times\left[\psi_{\substack{e\\o}mn}^{(i)}(c,\,\boldsymbol{r})\hat{\boldsymbol{a}}\right]. \tag{12b}$$

The explicit forms of the spheroidal vector wave functions under the alternative spheroidal coordinates systems are given by Flammer in [40].

It is worth mentioning that the singularity of the Green's functions was a controversial issue in the late 1970's [43]. The focused point was on the exact representation of the irrotational DGFs, which was missing in the first edition of Tai's book [44]. Now, the issue of irrotational DGFs has been well resolved and is no longer the problem to the electromagnetics community. In this paper, the irrotational part of the Green's dyadic is found from a combination of two contributions: one of them taken directly from the unit delta dyadic and the other obtained from the first-order derivative of the Green's function at the discontinuity point at $\xi = \xi'$ [6, pp. 128–129, 154]. The total effects of the two parts make the present form of the irrotational contribution to the Green's dyadic.

IV. Scattering Green's Dyadics

Using the principle of scattering superposition, the DGF can be considered as the sum of the unbounded Green's dyadic in (11) and a scattering Green's dyadic to be determined. The Green's dyadic is, therefore, given by [6]

$$\overline{\boldsymbol{G}}_{EJ}^{(fs)}(\boldsymbol{r},\,\boldsymbol{r}') = \overline{\boldsymbol{G}}_{EJ0}(\boldsymbol{r},\,\boldsymbol{r}')\delta_{fs} + \overline{\boldsymbol{G}}_{EJs}^{(fs)}(\boldsymbol{r},\,\boldsymbol{r}') \tag{13}$$

where the scattering DGF $\overline{\boldsymbol{G}}_{EJs}^{(fs)}(\boldsymbol{r},\,\boldsymbol{r}')$ describes an additional contribution of the multiple reflection and transmission waves in the presence of the boundary produced by the dielectric media, while the unbounded DGF $\overline{\boldsymbol{G}}_{EJ0}(\boldsymbol{r},\,\boldsymbol{r}')$, given by (11), represents the contribution of the direct waves from radiation sources in an unbounded medium. The subscript s identifies the scattering DGFs.

When the antenna is located in the sth region, the scattering DGF in the fth regions must be of the form similar to that of the unbounded Green's dyadic. To satisfy the boundary conditions, however, the additional spheroidal vector wave functions $\boldsymbol{M}_{\substack{o\\e}mn}^{a(i)}(c,\,\xi)$ should be included to account for the effects of multiple transmissions and reflections. For the ease of determination of the scattering coefficients, the sets of vector wave functions, $\boldsymbol{M}_{\substack{e\\o}m\pm1,n}^{\pm(1)}(c,\,\xi)$ and $\boldsymbol{N}_{\substack{e\\o}m\pm1,n}^{\pm(i)}(c,\,\xi)$ are used in the construction of the scattering DGFs. $\boldsymbol{M}_{\substack{e\\o}m\pm1,n}^{\pm(i)}(c,\,\xi)$ and $\boldsymbol{N}_{\substack{e\\o}m\pm1,n}^{\pm(i)}(c,\,\xi)$ are defined as follows:

$$\boldsymbol{X}_{\substack{e\\o}m+1,n}^{+(i)}(c,\,\xi) = \frac{1}{2}\left[\boldsymbol{X}_{\substack{e\\o}mn}^{x(i)}(c,\,\xi) \mp \boldsymbol{X}_{\substack{o\\e}mn}^{y(i)}(c,\,\xi)\right] \tag{14a}$$

$$\boldsymbol{X}_{\substack{e\\o}m-1,n}^{-(i)}(c,\,\xi) = \frac{1}{2}\left[\boldsymbol{X}_{\substack{e\\o}mn}^{x(i)}(c,\,\xi) \pm \boldsymbol{X}_{\substack{o\\e}mn}^{y(i)}(c,\,\xi)\right] \tag{14b}$$

where $\boldsymbol{X}$ denotes either $\boldsymbol{M}$ or $\boldsymbol{N}$.

For a two-layer spheroidal geometry, the DGFs have been given by Li *et al.* [39], [45]. Therefore, the scattering DGFs in each region of a multilayered spheroidal structure can be formulated in the following similar fashion:

$$
\begin{aligned}
&\overline{\boldsymbol{G}}^{(fs)}_{EJs}(\boldsymbol{r}, \boldsymbol{r}') \\
&\quad = \frac{i}{2\pi} \sum_{n=m}^{\infty} \sum_{m=0}^{\infty} (1-\delta_{fN}) \\
&\qquad \cdot \Big\{ \Big[\mathcal{A}^{+xM}_{f\substack{o\\e}mn} \boldsymbol{M}^{+(3)}_{\substack{o\\e}m+1,n}(c_f, \xi) + \mathcal{A}^{+xN}_{f\substack{e\\o}mn} \boldsymbol{N}^{+(3)}_{\substack{e\\o}m+1,n}(c_f, \xi) \\
&\qquad\quad + \mathcal{A}^{-xM}_{f\substack{o\\e}mn} \boldsymbol{M}^{-(3)}_{\substack{o\\e}m-1,n}(c_f, \xi) + \mathcal{A}^{-xN}_{f\substack{e\\o}mn} \boldsymbol{N}^{-(3)}_{\substack{e\\o}m-1,n}(c_f, \xi) \Big] \hat{\boldsymbol{x}} \\
&\qquad\quad + \Big[\mathcal{A}^{+yM}_{f\substack{e\\o}mn} \boldsymbol{M}^{+(3)}_{\substack{e\\o}m+1,n}(c_f, \xi) + \mathcal{A}^{+yN}_{f\substack{o\\e}mn} \boldsymbol{N}^{+(3)}_{\substack{o\\e}m+1,n}(c_f, \xi) \\
&\qquad\quad + \mathcal{A}^{-yM}_{f\substack{e\\o}mn} \boldsymbol{M}^{-(3)}_{\substack{e\\o}m-1,n}(c_f, \xi) + \mathcal{A}^{-yN}_{f\substack{o\\e}mn} \boldsymbol{N}^{-(3)}_{\substack{o\\e}m-1,n}(c_f, \xi) \Big] \hat{\boldsymbol{y}} \\
&\qquad\quad + \Big[\mathcal{A}^{zM}_{f\substack{o\\e}mn} \boldsymbol{M}^{z(3)}_{\substack{o\\e}mn}(c_f, \xi) + \mathcal{A}^{zN}_{f\substack{e\\o}mn} \boldsymbol{N}^{z(3)}_{\substack{e\\o}mn}(c_f, \xi) \Big] \hat{\boldsymbol{z}} \Big\} \\
&\qquad + (1-\delta_{f1}) \\
&\qquad \cdot \Big\{ \Big[\mathcal{B}^{+xM}_{f\substack{o\\e}mn} \boldsymbol{M}^{+(1)}_{\substack{o\\e}m+1,n}(c_f, \xi) + \mathcal{B}^{+xN}_{f\substack{e\\o}mn} \boldsymbol{N}^{+(1)}_{\substack{e\\o}m+1,n}(c_f, \xi) \\
&\qquad\quad + \mathcal{B}^{-xM}_{f\substack{o\\e}mn} \boldsymbol{M}^{-(1)}_{\substack{o\\e}m-1,n}(c_f, \xi) + \mathcal{B}^{-xN}_{f\substack{e\\o}mn} \boldsymbol{N}^{-(1)}_{\substack{e\\o}m-1,n}(c_f, \xi) \Big] \hat{\boldsymbol{x}} \\
&\qquad\quad + \Big[\mathcal{B}^{+yM}_{f\substack{e\\o}mn} \boldsymbol{M}^{+(1)}_{\substack{e\\o}m+1,n}(c_f, \xi) + \mathcal{B}^{+yN}_{f\substack{o\\e}mn} \boldsymbol{N}^{+(1)}_{\substack{o\\e}m+1,n}(c_f, \xi) \\
&\qquad\quad + \mathcal{B}^{-yM}_{f\substack{e\\o}mn} \boldsymbol{M}^{-(1)}_{\substack{e\\o}m-1,n}(c_f, \xi) + \mathcal{B}^{-yN}_{f\substack{o\\e}mn} \boldsymbol{N}^{-(1)}_{\substack{o\\e}m-1,n}(c_f, \xi) \Big] \hat{\boldsymbol{y}} \\
&\qquad\quad + \Big[\mathcal{B}^{zM}_{f\substack{o\\e}mn} \boldsymbol{M}^{z(1)}_{\substack{o\\e}mn}(c_f, \xi) + \mathcal{B}^{zN}_{f\substack{e\\o}mn} \boldsymbol{N}^{z(1)}_{\substack{e\\o}mn}(c_f, \xi) \Big] \hat{\boldsymbol{z}} \Big\}.
\end{aligned} \tag{15}
$$

Here, δ_{fN} and δ_{f1} are Kronecker delta functions. $c_s = (1/2)k_s d$ and $c_f = (1/2)k_f d$, where k_s and k_f are, respectively, the wave propagation constants in which the source and field points are located. $\mathcal{A}^{(\pm x, \pm y, z)M}_{f\substack{o\\e}mn}$, $\mathcal{A}^{(\pm x, \pm y, z)N}_{f\substack{e\\o}mn}$, $\mathcal{B}^{(\pm x, \pm y, z)M}_{f\substack{e\\o}mn}$, and $\mathcal{B}^{(\pm x, \pm y, z)N}_{f\substack{e\\o}mn}$ are unknown scattering coefficients to be determined from the boundary conditions.

V. Nonorthogonality and Item Expansion

After the substitution of (13) into (5a) and (5b), respectively, the following relations of vector wave functions are used in the vector operations:

$$\nabla \times \boldsymbol{M}^{\pm(i)}_{\substack{e\\o}m\pm1,n}(c, \xi) = k\boldsymbol{N}^{\pm(i)}_{\substack{e\\o}m\pm1,n}(c, \xi) \tag{16a}$$

$$\nabla \times \boldsymbol{M}^{z(i)}_{\substack{e\\o}mn}(c, \xi) = k\boldsymbol{N}^{z(i)}_{\substack{e\\o}mn}(c, \xi) \tag{16b}$$

$$\nabla \times \boldsymbol{N}^{\pm(i)}_{\substack{e\\o}m\pm1,n}(c, \xi) = k\boldsymbol{M}^{\pm(i)}_{\substack{e\\o}m\pm1,n}(c, \xi) \tag{16c}$$

$$\nabla \times \boldsymbol{N}^{z(i)}_{\substack{e\\o}mn}(c, \xi) = k\boldsymbol{M}^{z(i)}_{\substack{e\\o}mn}(c, \xi). \tag{16d}$$

These relations are the same as those of vector wave functions in the orthogonal coordinate systems [6], [40].

Due to the orthogonality of the trigonometric functions, the coefficients of the same ϕ-dependent trigonometric function in (5a) and (5b) must be equal, component by component; the equalities must hold for each corresponding term in the summation over m. For the summation over n, however, the individual terms in the series cannot be decomposed term by term because of the nonorthogonality of the spheroidal radial functions. This causes the difficulty in determining the unknown scattering coefficients.

To solve for the unknown coefficients, the following expanded intermediate forms [1], [2] are introduced:

for $m \geq 1$:

$$\sum_{t=0}^{\infty} I^{mn}_{t,l}(c) P^{m-1}_{m-1+t}(\eta) = (1-\eta^2)^{l-(1/2)} S^m_n(c, \eta), \qquad l = 0, 1, 2, 3 \tag{17a}$$

$$\sum_{t=0}^{\infty} I^{mn}_{t,4+l}(c) P^{m-1}_{m-1+t}(\eta) = \eta(1-\eta^2)^{l-(1/2)} S^m_n(c, \eta), \qquad l = 0, 1, 2 \tag{17b}$$

$$\sum_{t=0}^{\infty} I^{mn}_{t,6+l}(c) P^{m-1}_{m-1+t}(\eta) = (1-\eta^2)^{l-(1/2)} \frac{S^m_n(c, \eta)}{d\eta}, \qquad l = 1, 2, 3 \tag{17c}$$

$$\sum_{t=0}^{\infty} I^{mn}_{t,9+l}(c) P^{m-1}_{m-1+t}(\eta) = \eta(1-\eta^2)^{l-(1/2)} \frac{dS^m_n(c, \eta)}{d\eta}, \qquad l = 1, 2 \tag{17d}$$

and for $m = 0$:

$$\sum_{t=0}^{\infty} I^{0n}_{t,l}(c) P^{1}_{1+t}(\eta) = (1-\eta^2)^{l-(1/2)} S^0_n(c, \eta), \qquad l = 1, 2, 3 \tag{18a}$$

$$\sum_{t=0}^{\infty} I^{0n}_{t,4+l}(c) P^{1}_{1+t}(\eta) = \eta(1-\eta^2)^{l-(1/2)} S^0_n(c, \eta), \qquad l = 1, 2 \tag{18b}$$

$$\sum_{t=0}^{\infty} I^{0n}_{t,6+l}(c) P^{1}_{1+t}(\eta) = (1-\eta^2)^{l-(1/2)} \frac{dS^0_n(c, \eta)}{d\eta}, \qquad l = 1, 2, 3 \tag{18c}$$

$$\sum_{t=0}^{\infty} I^{0n}_{t,9+l}(c) P^{1}_{1+t}(\eta) = \eta(1-\eta^2)^{l-(1/2)} \frac{dS^0_n(c, \eta)}{d\eta}, \qquad l = 1, 2 \tag{18d}$$

where $S^m_n(c, \eta)$ and $S^0_n(c, \eta)$ are spheroidal angular functions, $P^{m-1}_{m-1+t}(\eta)$ and $P^1_{1+t}(\eta)$ are associate Legendre functions, and $I^{mn}_{t,\ell}$ $(t = 0, 1, 2, \ldots)$ are intermediates, which have been provided in [46] and [47]. The individual terms in the summation over t must be matched term by term, by considering the orthogonality of $P^{m-1}_{m-1+t}(\eta)$ and $P^1_{1+t}(\eta)$. By substitution of the above equations, all factors being functions of η are replaced by a series of the associated Legendre functions, which are orthogonal functions in the interval $-1 \leq \eta \leq 1$.

VI. Matrix Equation Systems

Finally, the equations used to determine the unknown coefficients constitute an infinite system of coupled linear equations and the unknown coefficients can be solved for from the following matrix equation systems:

$$\left(\boldsymbol{\Lambda}^h_{\mathcal{AB}}\right) = \left(\boldsymbol{\Delta}^h_{\mathcal{UV}}\right)^{-1} \cdot \left(\boldsymbol{\Upsilon}^h\right) \cdot \left(\boldsymbol{\Gamma}^h\right)$$
$$h = +x,\ -x,\ +y,\ -y,\ \text{and } z,\ \text{respectively.} \tag{19}$$

Here, $(\boldsymbol{\Lambda}^h_{\mathcal{AB}})$ is the matrix of the unknown coefficients to be determined, $(\boldsymbol{\Gamma}^h)$ is the constant matrix in which the integrals of source currents are involved, and $(\boldsymbol{\Delta}^h_{\mathcal{UV}})$ and $(\boldsymbol{\Upsilon}^h)$ are the matrices of constant elements obtained from the functional expansions. For an N-layered spheroidal structure, if the truncation number of the summation over n is chosen as N_T, which means that n is taken as $m,\ m+1,\ \ldots,\ m+N_T-1$ as an approximation to the infinite summation for a given m, the matrices in (19) can be expressed subsequently.

A. Matrix $(\boldsymbol{\Lambda}^h_{\mathcal{AB}})$

The matrix in the left-hand side of (19) is found in its explicit form to be

$$\left(\boldsymbol{\Lambda}^h_{\mathcal{AB}}\right) = \begin{pmatrix} \mathbf{A}^h_1 \\ \mathbf{A}^h_2 \\ \vdots \\ \mathbf{A}^h_{N-1} \\ \mathbf{B}^h_2 \\ \mathbf{B}^h_3 \\ \vdots \\ \mathbf{B}^h_N \end{pmatrix} \tag{20}$$

where the element matrices are defined as

$$\mathbf{A}^h_f = \begin{pmatrix} \mathcal{A}^{hM}_{f\,\substack{o\\e}\,m,m} \\ \mathcal{A}^{hM}_{f\,\substack{o\\e}\,m,m+1} \\ \vdots \\ \mathcal{A}^{hM}_{f\,\substack{o\\e}\,m,m+N_T-1} \\ \mathcal{A}^{hN}_{f\,\substack{e\\o}\,m,m} \\ \mathcal{A}^{hN}_{f\,\substack{e\\o}\,m,m+1} \\ \vdots \\ \mathcal{A}^{hN}_{f\,\substack{e\\o}\,m,m+N_T-1} \end{pmatrix}, \qquad \text{for } f = 1,\ 2,\ 3,\ \ldots,\ N-1$$

and

$$\mathbf{B}^h_f = \begin{pmatrix} \mathcal{B}^{hM}_{f\,\substack{o\\e}\,m,m} \\ \mathcal{B}^{hM}_{f\,\substack{o\\e}\,m,m+1} \\ \vdots \\ \mathcal{B}^{hM}_{f\,\substack{o\\e}\,m,m+N_T-1} \\ \mathcal{B}^{hN}_{f\,\substack{e\\o}\,m,m} \\ \mathcal{B}^{hN}_{f\,\substack{e\\o}\,m,m+1} \\ \vdots \\ \mathcal{B}^{hN}_{f\,\substack{e\\o}\,m,m+N_T-1} \end{pmatrix}, \qquad \text{for } f = 2,\ 3,\ 4,\ \ldots,\ N.$$

B. Matrix $(\boldsymbol{\Delta}^h)$

In the similar fashion, the first matrix in the right-hand side of (19) is expressed explicitly as

$$\left(\boldsymbol{\Delta}^h\right) = \begin{pmatrix} \boldsymbol{\Omega}^h_0 \\ \boldsymbol{\Omega}^h_1 \\ \vdots \\ \boldsymbol{\Omega}^h_{m+N_T-1} \end{pmatrix} \tag{21a}$$

where the element matrices are given in (21b), shown at the bottom of the following page, where $\mathbf{0}$ is the zero matrix; the sub-matrices $\mathbf{W}^f_f$ are given for $f = 1$ by

$$\mathbf{W}^f_f = \begin{pmatrix} \left(\mathbf{U}^{h(3),\,t}_{\eta}\right)^1_1 & -\left(\mathbf{V}^{h(3),\,t}_{\eta}\right)^1_1 \\ \left(\mathbf{U}^{h(3),\,t}_{\phi}\right)^1_1 & -\left(\mathbf{V}^{h(3),\,t}_{\phi}\right)^1_1 \\ \left(\mathbf{V}^{h(3),\,t}_{\eta}\right)^1_1 & -\left(\mathbf{U}^{h(3),\,t}_{\eta}\right)^1_1 \\ \left(\mathbf{V}^{h(3),\,t}_{\phi}\right)^1_1 & -\left(\mathbf{U}^{h(3)}_{\phi}\right)^1_1 \end{pmatrix}$$

and for $2 \le f \le N-1$ by

$$\mathbf{W}^f_f = \begin{pmatrix} \left(\mathbf{U}^{h(3),\,t}_{\eta}\right)^f_f & \left(\mathbf{V}^{h(3),\,t}_{\eta}\right)^f_f & \left(\mathbf{U}^{h(1),\,t}_{\eta}\right)^f_f & \left(\mathbf{V}^{h(1),\,t}_{\eta}\right)^f_f \\ \left(\mathbf{U}^{h(3),\,t}_{\phi}\right)^f_f & \left(\mathbf{V}^{h(3),\,t}_{\phi}\right)^f_f & \left(\mathbf{U}^{h(1),\,t}_{\phi}\right)^f_f & \left(\mathbf{V}^{h(1),\,t}_{\phi}\right)^f_f \\ \left(\mathbf{V}^{h(3),\,t}_{\eta}\right)^f_f & \left(\mathbf{U}^{h(3),\,t}_{\eta}\right)^f_f & \left(\mathbf{V}^{h(1),\,t}_{\eta}\right)^f_f & \left(\mathbf{U}^{h(1),\,t}_{\eta}\right)^f_f \\ \left(\mathbf{V}^{h(3),\,t}_{\phi}\right)^f_f & \left(\mathbf{U}^{h(3),\,t}_{\phi}\right)^f_f & \left(\mathbf{V}^{h(1),\,t}_{\phi}\right)^f_f & \left(\mathbf{U}^{h(1),\,t}_{\phi}\right)^f_f \end{pmatrix}$$

and the sub-matrices $\mathbf{W}^f_{f-1}$ are derived for $f = N$ as

$$\mathbf{W}^f_{f-1} = \begin{pmatrix} -\left(\mathbf{U}^{h(1),\,t}_{\eta}\right)^N_{N-1} & -\left(\mathbf{V}^{h(1),\,t}_{\eta}\right)^N_{N-1} \\ -\left(\mathbf{U}^{h(1),\,t}_{\phi}\right)^N_{N-1} & -\left(\mathbf{V}^{h(1),\,t}_{\phi}\right)^N_{N-1} \\ -\rho_N\left(\mathbf{V}^{h(1),\,t}_{\eta}\right)^N_{N-1} & -\rho_N\left(\mathbf{U}^{h(1),\,t}_{\eta}\right)^N_{N-1} \\ -\rho_N\left(\mathbf{V}^{h(1),\,t}_{\phi}\right)^N_{N-1} & -\rho_N\left(\mathbf{U}^{h(1)}_{\phi}\right)^N_{N-1} \end{pmatrix}$$

and for $2 \le f \le N-1$ as

$$\mathbf{W}^f_{f-1} = \begin{pmatrix} -\left(\mathbf{U}^{h(3),\,t}_{\eta}\right)^f_{f-1} & -\left(\mathbf{V}^{h(3),\,t}_{\eta}\right)^f_{f-1} \\ -\left(\mathbf{U}^{h(3),\,t}_{\phi}\right)^f_{f-1} & -\left(\mathbf{V}^{h(3),\,t}_{\phi}\right)^f_{f-1} \\ -\rho_f\left(\mathbf{V}^{h(3),\,t}_{\eta}\right)^f_{f-1} & -\rho_f\left(\mathbf{U}^{h(3),\,t}_{\eta}\right)^f_{f-1} \\ -\rho_f\left(\mathbf{V}^{h(3),\,t}_{\phi}\right)^f_{f-1} & -\rho_f\left(\mathbf{U}^{h(3),\,t}_{\phi}\right)^f_{f-1} \end{pmatrix}$$

$$\left.\begin{matrix} -\left(\mathbf{U}_\eta^{h(1),t}\right)_{f-1}^{f} & -\left(\mathbf{V}_\eta^{h(1),t}\right)_{f-1}^{f} \\ -\left(\mathbf{U}_\phi^{h(1),t}\right)_{f-1}^{f} & -\left(\mathbf{V}_\phi^{h(1),t}\right)_{f-1}^{f} \\ -\rho_f\left(\mathbf{V}_\eta^{h(3),t}\right)_{f-1}^{f} & -\rho_f\left(\mathbf{U}_\eta^{h(3),t}\right)_{f-1}^{f} \\ -\rho_f\left(\mathbf{V}_\phi^{h(3),t}\right)_{f-1}^{f} & -\rho_f\left(\mathbf{U}_\phi^{h(3),t}\right)_{f-1}^{f} \end{matrix}\right).$$

In the above equations, $\rho_f = \sqrt{\epsilon_f/\mu_f}/\sqrt{\epsilon_{f-1}/\mu_{f-1}}$. Details of $(\mathbf{U}_\eta^{h(i),t})_q^p$, $(\mathbf{U}_\phi^{h(i),t})_q^p$, $(\mathbf{V}_\eta^{h(i),t})_q^p$, and $(\mathbf{V}_\phi^{h(i),t})_q^p$ (here $p = f$, $q = f$ or $f-1$, and f denotes the location of the field point) in different cases are provided in the Appendix.

C. Matrix $(\boldsymbol{\Upsilon}^h)$

The second matrix in the right-hand side of (19) is expressed as

$$\left(\boldsymbol{\Upsilon}^h\right) = \begin{pmatrix} \mathbf{C}_0^h \\ \mathbf{C}_1^h \\ \vdots \\ \mathbf{C}_{m+N_T-1}^h \end{pmatrix}. \tag{22a}$$

where the element matrices are defined for the case where the source is located in the out region ($s = 1$) as

$$\mathbf{C}_t^h = \begin{pmatrix} \left(\mathbf{V}_\eta^{h(1),t}\right)_1^1 \\ \left(\mathbf{V}_\phi^{h(1),t}\right)_1^1 \\ \left(\mathbf{U}_\eta^{h(1),t}\right)_1^1 \\ \left(\mathbf{U}_\phi^{h(1),t}\right)_1^1 \\ \mathbf{0} \end{pmatrix} \tag{22b}$$

for the case where the source is located in one of the intermediate layers ($2 \le s \le N-1$) as

$$\mathbf{C}_t^h = \begin{pmatrix} \mathbf{0} \\ \left(\mathbf{V}_\eta^{h(3),t}\right)_{s-1}^{s} \\ \left(\mathbf{V}_\phi^{h(3),t}\right)_{s-1}^{s} \\ \rho_s\left(\mathbf{U}_\eta^{h(3),t}\right)_{s-1}^{s} \\ \rho_s\left(\mathbf{U}_\phi^{h(3),t}\right)_{s-1}^{s} \\ \left(\mathbf{V}_\eta^{h(1),t}\right)_{s}^{s} \\ \left(\mathbf{V}_\phi^{h(1),t}\right)_{s}^{s} \\ \left(\mathbf{U}_\eta^{h(1),t}\right)_{s}^{s} \\ \left(\mathbf{U}_\phi^{h(1),t}\right)_{s}^{s} \\ \mathbf{0} \end{pmatrix} \tag{22c}$$

and for the case where the source is located in the inner region ($s = N$) as

$$\mathbf{C}_t^h = \begin{pmatrix} \mathbf{0} \\ \left(\mathbf{V}_\eta^{h(3),t}\right)_{N-1}^{N} \\ \left(\mathbf{V}_\phi^{h(3),t}\right)_{N-1}^{N} \\ \rho_N\left(\mathbf{U}_\eta^{h(3),t}\right)_{N-1}^{N} \\ \rho_N\left(\mathbf{U}_\phi^{h(3),t}\right)_{N-1}^{N} \end{pmatrix}.$$

In the above equations, $\mathbf{0}$ is the zero matrix, $\rho_s = \sqrt{\epsilon_s/\mu_s}/\sqrt{\epsilon_{s-1}/\mu_{s-1}}$. Details of $(\mathbf{U}_\eta^{h(i),t})_q^p$, $(\mathbf{U}_\phi^{h(i),t})_q^p$, $(\mathbf{V}_\eta^{h(i),t})_q^p$, and $(\mathbf{V}_\phi^{h(i),t})_q^p$ (here, $p = s$, $q = s$, or $s-1$, and s denotes the location of the source point) in different cases are provided in the Appendix.

$$\boldsymbol{\Omega}_t^h = \begin{pmatrix} \mathbf{W}_1^1 & \mathbf{W}_1^2 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_2^2 & \mathbf{W}_2^3 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{W}_{f-1}^{f-1} & \mathbf{W}_{f-1}^{f} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{W}_{f}^{f} & \mathbf{W}_{f}^{f+1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{W}_{N-2}^{N-2} & \mathbf{W}_{N-2}^{N-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{W}_{N-1}^{N-1} & \mathbf{W}_{N-1}^{N} \end{pmatrix} \tag{21b}$$

D. Matrix $(\mathbf{\Gamma}^h)$

The last matrix employed in the right-hand side of (19) is defined as follows. In the case that the source located in the outer region ($s = 1$)

$$(\mathbf{\Gamma}^h) = \begin{pmatrix} \mathbf{\Psi}^{h(1)}_{\substack{e\\o}} \\ \mathbf{0} \end{pmatrix}, \qquad \text{for } h = +x,\, -x,\, z \tag{23a}$$

and

$$(\mathbf{\Gamma}^h) = \begin{pmatrix} \mathbf{\Psi}^{h(1)}_{\substack{o\\e}} \\ \mathbf{0} \end{pmatrix}, \qquad \text{for } h = +y,\, -y. \tag{23b}$$

In the case that the source is located in the intermediate region ($2 \le s \le N - 1$)

$$(\mathbf{\Gamma}^h) = \begin{pmatrix} \mathbf{0} \\ \mathbf{\Psi}^{h(1)}_{\substack{e\\o}} \\ \mathbf{\Psi}^{h(3)}_{\substack{e\\o}} \\ \mathbf{0} \end{pmatrix}, \qquad \text{for } h = +x,\, -x,\, z \tag{23c}$$

and

$$(\mathbf{\Gamma}^h) = \begin{pmatrix} \mathbf{0} \\ \mathbf{\Psi}^{h(1)}_{\substack{o\\e}} \\ \mathbf{\Psi}^{h(3)}_{\substack{o\\e}} \\ \mathbf{0} \end{pmatrix}, \qquad \text{for } h = +y,\, -y. \tag{23d}$$

In the case that the source is located in the inner region ($s = N$)

$$(\mathbf{\Gamma}^h) = \begin{pmatrix} \mathbf{0} \\ \mathbf{\Psi}^{h(3)}_{\substack{e\\o}} \end{pmatrix}, \qquad \text{for } h = +x,\, -x,\, z \tag{23e}$$

and

$$(\mathbf{\Gamma}^h) = \begin{pmatrix} \mathbf{0} \\ \mathbf{\Psi}^{h(3)}_{\substack{o\\e}} \end{pmatrix}, \qquad \text{for } h = +y,\, -y \tag{23f}$$

where "0" is the zero matrix and

$$\mathbf{\Psi}^{h(i)}_{\substack{e\\o}} = \begin{pmatrix} \dfrac{2-\delta_{m0}}{N_{mm}}\,\psi^{(i)}_{\substack{e\\o}mm}(c_s,\,\xi') \\ \dfrac{2-\delta_{m0}}{N_{m(m+1)}}\,\psi^{(i)}_{\substack{e\\o}m(m+1)}(c_s,\,\xi') \\ \vdots \\ \dfrac{2-\delta_{m0}}{N_{m(m+N_T-1)}}\,\psi^{(i)}_{\substack{e\\o}m(m+N_T-1)}(c_s,\,\xi') \end{pmatrix}$$

$$\mathbf{\Psi}^{h(i)}_{\substack{o\\e}} = \begin{pmatrix} \dfrac{2-\delta_{m0}}{N_{mm}}\,\psi^{(i)}_{\substack{o\\e}mm}(c_s,\,\xi') \\ \dfrac{2-\delta_{m0}}{N_{m(m+1)}}\,\psi^{(i)}_{\substack{o\\e}m(m+1)}(c_s,\,\xi') \\ \vdots \\ \dfrac{2-\delta_{m0}}{N_{m(m+N_T-1)}}\,\psi^{(i)}_{\substack{o\\e}m(m+N_T-1)}(c_s,\,\xi') \end{pmatrix}.$$

In (20)–(23f), for each specified t value, each $\mathbf{\Omega}_t^h$ or $\mathbf{C}_t^h$ contains $4(N-1)$ rows for an N-layered spheroidal structure. Therefore, there are $4(N-1)$ sets of unknowns that are of infinite number and coupled to each other [the η- and ϕ-components derived from (5a) and (5b)]. In principle, by making t sufficiently large, an adequate number of relations satisfied by unknown coefficients are formulated and the unknown coefficients can be determined explicitly. However, it is indicated by Asano and Yamamoto [1] that a sufficient computational accuracy can be achieved by choosing a proper truncation number that can, as suggested by Sinha and MacPhie [2], be taken as $N_T = \text{Integer}(|c|+4)$. In other words, there exit $4(N-1)\times N_T$ scalar unknowns (or N_T vector unknowns) for each of the aforementioned equations. Assuming that the index t in the equations is taken as $0,\, 1,\, \ldots,\, N_T - 1$, we can determine these $4(N-1)\times N_T$ scalar unknowns (or the N_T vector unknowns) uniquely.

VII. Discussions and Conclusion

In this paper, the DGFs in multilayered spheroidal structures have been formulated in terms of these appropriate spheroidal vector wave eigenfunctions ($\boldsymbol{M}^{\pm(i)}_{\substack{e\\o}m\pm1,n}$, $\boldsymbol{M}^{z(i)}_{\substack{e\\o}mn}$, $\boldsymbol{N}^{\pm(i)}_{\substack{e\\o}m\pm1,n}$, and $\boldsymbol{N}^{z(i)}_{\substack{e\\o}mn}$) and coordinate vectors $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{z}}$. The representation and formulation of DGFs in the spheroidal coordinates have been made in a different eigenfunction expansion, as compared with those conventional formulations in planar, cylindrical, and spherical coordinates. The nonsolenoidal term of the electric DGF has been extracted by following the same procedure given by Tai [6]. The unknown coefficients of the scattering DGFs can, even coupled to each other, be determined from the matrix equation systems using the functional expansion technique. The scattering coefficients of the Green's dyadic for various cases of source and field locations have been provided. Although the determination of these matrix-formed coefficients has been provided in close form in this paper, it should be noted here that the integrals of source currents are involved in $(\mathbf{\Gamma}^h)$ of (23a)–(23f) and the unknowns cannot be obtained analytically from the equations without the integrals of source currents. This is different from the spherical case, where the scattering coefficients are independent of integrals of source currents and decoupled from each other. However, for a given source excitation in a spheroidal structure, the distribution is known and the source point does not appear after the integration over the whole source region.

Besides the lack of orthogonality of spheroidal vector wave functions, the very complicated calculation of the spheroidal angular and radial harmonics is another difficulty in obtaining the analytical solutions in spheroidal structures. With the development of computer facilities and the appropriate method and routine of calculation [48], the tabulated values of spheroidal angular and radial functions can be easily obtained and more analytical solutions in spheroidal structures can be found directly by using the presented Green's dyadic. Applications of the DGFs in spheroidal structures presented here can be found from many practical problems such as the EM waves inside and outside a stratified prolate dielectric radome utilized to protect airborne or satellite antennas from the environmental effects [49], handy phone radiation near the layered spheroid-shaped human head [46], [47], and rainfall attenuation of microwave signals due to oblate raindrops [50]. Some numerical results about the applications of the formulated DGFs will be presented in part II of this paper.

APPENDIX

Subsequently provided are the detailed expressions of $(\mathbf{U}_\eta^{h(i),t})_q^p$, $(\mathbf{U}_\phi^{h(i),t})_q^p$, $(\mathbf{V}_\eta^{h(i),t})_q^p$, and $(\mathbf{V}_\phi^{h(i),t})_q^p$ (where $h = +x, -x, +y, -y$ and z, respectively, and $i = 1$ and 3) involved in (21a)–(22c)

$$\left(\mathbf{U}_\eta^{h(i),t}\right)_q^p = \Big(\mathcal{U}^{h'(i),t}_{\substack{e\\o}\eta mm}(c_p, \xi_q)\ \mathcal{U}^{h'(i),t}_{\substack{e\\o}\eta m(m+1)}(c_p, \xi_q) \ \ldots\ \mathcal{U}^{h'(i),t}_{\substack{e\\o}\eta m(m+N_T-1)}(c_p, \xi_q)\Big) \quad \text{(A-1a)}$$

$$\left(\mathbf{U}_\phi^{h(i),t}\right)_q^p = \Big(\mathcal{U}^{h'(i),t}_{\substack{e\\o}\phi mm}(c_p, \xi_q)\ \mathcal{U}^{h'(i),t}_{\substack{e\\o}\phi m(m+1)}(c_p, \xi_q) \ \ldots\ \mathcal{U}^{h'(i),t}_{\substack{e\\o}\phi m(m+N_T-1)}(c_p, \xi_q)\Big) \quad \text{(A-1b)}$$

$$\left(\mathbf{V}_\eta^{h(i),t}\right)_q^p = \Big(\mathcal{V}^{h'(i),t}_{\substack{e\\o}\eta mm}(c_p, \xi_q)\ \mathcal{V}^{h'(i),t}_{\substack{e\\o}\eta m(m+1)}(c_p, \xi_q) \ \ldots\ \mathcal{V}^{h'(i),t}_{\substack{e\\o}\eta m(m+N_T-1)}(c_p, \xi_q)\Big) \quad \text{(A-1c)}$$

$$\left(\mathbf{V}_\phi^{h(i),t}\right)_q^p = \Big(\mathcal{V}^{h'(i),t}_{\substack{e\\o}\phi mm}(c_p, \xi_q)\ \mathcal{V}^{h'(i),t}_{\substack{e\\o}\phi m(m+1)}(c_p, \xi_q) \ \ldots\ \mathcal{V}^{h'(i),t}_{\substack{e\\o}\phi m(m+N_T-1)}(c_p, \xi_q)\Big). \quad \text{(A-1d)}$$

Here, $h' = +$ for $h = +x$ and $h = +y$, $h' = -$ for $h = -x$ and $h = -y$, $h' = z$ for $h = z$. The closed forms of $\mathcal{U}^{h(i),t}_{\substack{e\\o}\eta mn}(c_p, \xi_q)$, $\mathcal{U}^{h(i),t}_{\substack{e\\o}\phi mn}(c_p, \xi_q)$, $\mathcal{V}^{h(i),t}_{\substack{e\\o}\eta mn}(c_p, \xi_q)$ and $\mathcal{V}^{h(i),t}_{\substack{e\\o}\phi mn}(c_p, \xi_q)$ are provided in [46] and [47].

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